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HEAT TRANSFER IN ENTRANCE-REGION FLOW WITH EXTERNAL RESISTANCE

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NOMENCLATURE

h, film heat transfer coefficient;
H, $K_\infty/K_{\text{entrance}}$;
k, thermal conductivity;
K, consistency coefficient in power-law expression;
n, exponent in power-law expression;
Nu, Nusselt number, hD/k;
Nu', equivalent overall Nusselt number, UD/k;
Nu'', equivalent external Nusselt number, D/Rk;
Pr, Prandtl number, $cK/k(r/u_m)^{1-n}$;
r, radius;
R, external thermal resistance (based on inside tube area);
Re', Reynolds number, $r^n u_m^{2-n}\rho/K$;
u, axial velocity;
U, overall heat transfer coefficient;
x, axial distance;
x_0, dimensionless distance, $x/rRe'Pr$.

Greek symbols
ρ, density;
τ, shear stress.

Subscripts
m, mean;
x, local;
∞, ambient or asymptotic.

MANY analyses have been made of entrance-region heat transfer with fluids flowing in circular tubes for cases of constant wall temperature and constant heat flux. A more realistic condition in many applications, however, will include a resistance such as a layer of insulation or an external fluid film between the fluid in the tube and the external heat transfer medium. The only previously published study related to this problem (Schenk and Dumoré [1]), was based on the condition of fully developed laminar flow of constant-property Newtonian fluids.

The present work considers the problem of heat transfer to a fluid entering a circular tube in laminar flow with uniform velocity and temperature, with a constant external resistance between the tube wall and the constant ambient temperature. This work, which is an extension of the previous study of McKillop *et al.* [2], includes the effects of temperature-dependent viscosity and non-Newtonian behavior that can be described by the power-law model, $\tau = K(\mathrm{d}u/\mathrm{d}r)^n$.

The overall thermal resistance at any downstream point in the tube is equal to the sum of the fluid and external resistances, or

$$1/U_x = 1/h_x + R$$

where U_x is the local overall heat transfer coefficient; h_x the fluid coefficient; and R the constant resistance of the tube wall, insulation, and external fluid film, based on the inside tube area. It is convenient to multiply the above expression through by the ratio k/D so as to convert all terms to equivalent Nusselt numbers.

$$1/Nu'_x = 1/Nu_x + 1/Nu''. \tag{1}$$

In equation (1), Nu_x is the usual fluid Nusselt number, Nu'_x an overall Nusselt number, and Nu'' a constant Nusselt number representing the external resistance. In the previous study [2], constant-property results were expressed in terms of the Prandtl number and the power-law exponent n as parameters; to these must now be added the constant Nu''. At the tube entrance, Nu_x is infinite and Nu'_x is therefore equal to Nu''. At the opposite extreme of fully developed flow, Nu_x and Nu'_x become independent of Pr and approach constant values that are dependent only on n and Nu''.

For design purposes, we desire a mean Nusselt number to calculate the total heat transferred for any specified tube length. A mean overall Nusselt number can be defined by the equation

$$Nu'_m = (1/L)\int_0^L Nu'_x\,\mathrm{d}x \tag{2}$$

where the value of Nu'_x is obtained from equation (1). With this definition the total heat transferred from the entrance up to a length of tube L is given by

$$q = \pi kLNu'_m\Delta t_{1m} \tag{3}$$

where Δt_{1m} is the logarithmic mean of the overall temperature differences at the entrance and at distance L.

Solutions were obtained with the finite difference method of the previous study except that a boundary condition specifying the wall temperature gradient in terms of Nu'' replaced the constant wall-temperature boundary condition. The errors of that solution in the immediate vicinity of the entrance are of minor importance in this application because heat transfer rates in this region are determined primarily by the external resistance. Results presented below for Nu'' of infinity, corresponding to zero external resistance, were taken from the previous study, which used an integral boundary-layer solution in the immediate entrance region.

RESULTS AND DISCUSSION

Constant property

Figure 1 shows typical curves of Nu_x vs. x_0 for n of 1·0, Pr of 0·7 and various values of Nu''. The lower line in this figure, for $Nu'' = \infty$, simply corresponds to the case of zero external resistance and constant wall temperature. At the opposite limit of $Nu'' = 0$, the total resistance is infinite and the heat flux is therefore constant at zero. The condition of zero heat flux is simply the limiting case of constant heat flux operation, and curves for all intermediate values of Nu'' must therefore lie between the two limiting solutions of constant temperature and constant heat flux. In the immediate region of the entrance, Nu_x becomes very large and, the condition again exists that the total resistance is equal to the external resistance. Over a sufficiently short length of tube downstream from the entrance, the fluid temperature does not change appreciably, and the resulting condition of constant resistance and constant Δt corresponds to the constant heat flux case. The curves for all values of Nu'' must therefore approach the constant heat flux line as x approaches zero. The asymptotic values that Nu_x approaches at large x are dependent on Nu'' and are listed in Table 1.

Table 1. *Asymptotic values of* Nu_x

n	1·0	0·7	0·5
Nu''			
0	4·36	4·56	4·78
4	4·0	4·17	4·36
40	3·71	3·86	4·02
200	3·66	3·81	3·97
Infinite	3·66	3·80	3·95

The curves for various Nu_x can be correlated by the equation

$$\ln(Nu_x/Nu_\infty) = \sum_j a_j[\ln(1/x_0)]^j. \tag{4}$$

The coefficients a_j were determined by a least squares method and are listed for n of 1·0, 0·7 and 0·5 in a separate tabulation.* Equation (4) yields values of Nu_x with a maximum deviation of 2 per cent for $x_0 > 5 \times 10^{-4}$. Mean overall Nusselt numbers were calculated by applying equations (1) and (2) to values of Nu_x obtained from equation (4). These results are presented as a ratio of Nu'_m/Nu'_∞, also

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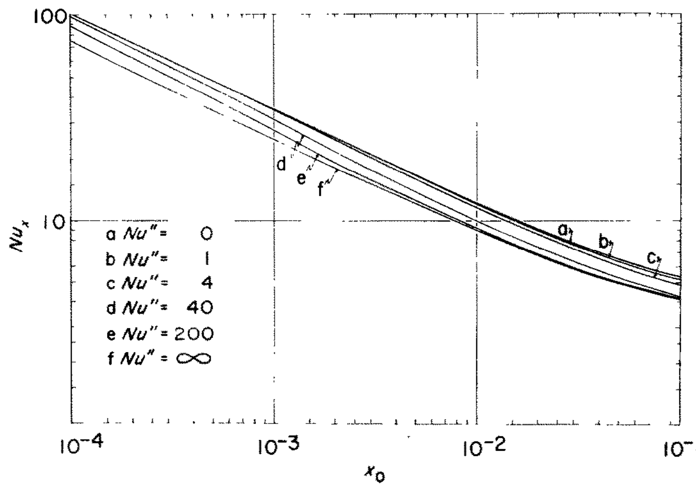

FIG. 1. Effect of external resistance on local fluid Nusselt number for $Pr = 0·7$ and $n = 1·0$.

tabulated separately. Over a large portion of entrance region flow, this ratio is substantially independent of n. The effect of n becomes more important at smaller x_0 and Pr and at larger Nu''. For each n and Nu'', curves for all Pr lie above the curve for fully developed entering velocity and approach it as x_0 increases. As Pr increases, velocity development requires a smaller portion of the total entrance region, and the Nusselt number curves are lower and approach the fully developed flow line at smaller values of x_0. Fully developed flow corresponds to the case of infinite Pr, where velocity development is completed before thermal development starts.

Variable viscosity

The previous study showed that with no external resistance, the effect of variable viscosity could be expressed in terms of a parameter H, which is the ratio of the consistency coefficient K (or viscosity for Newtonian fluids) at the wall temperature to that at the entering temperature. H is thus greater than 1·0 for cooling and less than 1·0 for heating. For a given value of H (or $1/H$), the decrease in Nu in cooling is about the same as the increase in heating. Heating and cooling lines for constant wall temperature parallel the constant property line at small values of x_0, and all lines have merged into the common asymptote for fully developed flow at x_0 of 1·0.

Similar qualitative effects are to be expected here. Figure 2 shows plots of Nu'_x for heating and cooling, for n of 1·0, Nu'' of 40, and Pr of 10 and 1000. In this figure, Pr is based on the ambient temperature, not the entering temperature. Since all lines therefore correspond to the same uniform temperature at the end of the entrance region, the asymptotic Nu'_∞ will not be affected by the value of H. As mentioned above for the previous study, the end of the entrance region where all lines have merged into the common asymptote corresponds to x_0 of about 1·0 regardless of the value of Pr or H. Furthermore, for a given velocity and Pr, any particular value of x_0 corresponds to the same physical tube length for all values of H.

In contrast to the constant wall temperature behavior of the previous study, the heating and cooling lines of Fig. 2 intersect. This behavior is a result of the fact that, with an appreciable external resistance, the wall temperature in the vicinity of the entrance is closer to the entering temperature than to the ambient temperature. Since the Nusselt number is established primarily by conditions near the wall, the effective Pr in the immediate entrance therefore corresponds more closely to the entering temperature than to the ambient temperature. Where there is no external resistance, the effective Pr corresponds more closely to the ambient (or wall) temperature. In cooling, for example, the fluid enters at a higher temperature and is cooled toward the ambient temperature. According to the above reasoning, the effective Pr is lower than the ambient Pr values used as a basis for Fig. 2. As pointed out in the discussion of constant property results, a lower value of Pr leads to a higher Nu in the entrance region. Therefore, the Nusselt number at any downstream distance (or x_0) will be higher than if Pr were constant throughout. At some sufficiently small x_0, this increase in Nu caused by the lower Pr is more than sufficient to offset the opposing effect of the reduced wall velocity gradient resulting from cooling of a fluid with a temperature-dependent viscosity. Analogous reasoning applies to the heating situation. As the wall temperature approaches the ambient temperature farther downstream, the expected velocity gradient effects become predominant, and the cooling curve falls below the heating curve.

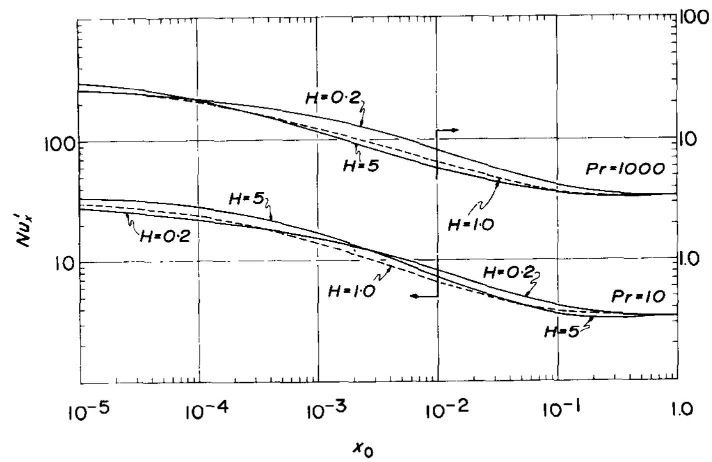

FIG. 2. Effect of variable viscosity on local overall Nusselt number for $Nu'' = 40$ and $n = 1·0$.

The presence of external resistance has a dual effect. By acting as a constant additive term in equation (1), it moderates the influence of changes in Nu_x on Nu'. Furthermore, it reduces temperature gradients in the fluid and thereby reduces the effects of heating and cooling. The values of H shown in Fig. 2 represent substantial degrees of heating and cooling, and yet the maximum spread between a pair of lines is only about 40 per cent. For Nu'' of 4, the corresponding spread is only about 5 per cent and can be neglected. In the opposite direction of increasing Nu'', the limit is set by the constant wall temperature case. The previous study indicated that deviations from constant property flow caused by the variable viscosity effects depend only to a minor extent on the value of n. The brief results presented here, therefore, when taken together with the previous study, provide an approximate basis for making adjustments to handle design problems with variable viscosity flow in the presence of an external resistance. From a practical consideration, uncertainties as to the rheological properties of a fluid and as to the correspondence of an actual system with the model used here could be of more importance than uncertainties in the adjustment for variable viscosity.

ACKNOWLEDGEMENT

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